Quantification and Correction of Signal Averaging with On-the-Fly Sampling in Near-Field Antenna Measurements

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Abstract— **In order to achieve an acceptable signal-to-noise ratio in practical near-field antenna measurements, probe signal averaging is usually employed. Hence, the nominal sample of the probe signal at the nominal sampling point is actually an average of multiple – between two and several millions - actual samples. For on-the-fly sampling, these actual samples are not taken at the same nominal sampling point but at different actual sampling points over a sampling interval – where the probe signal is not constant. If the nominal sample of the probe signal is treated as a point quantity in the subsequent post-processing an error is thus introduced. The present work shows that this erroneous effect of signal averaging with on-the-fly sampling can be exactly quantified and exactly corrected for.**

*Index Terms—*antenna measurements, near-field antenna measurements, signal averaging, on-the-fly sampling

I. INTRODUCTION

In near-field antenna measurements, the spatially band-limited probe signal is sampled at nominal sampling points distributed over a scan surface in front of, or enclosing, the antenna under test (AUT) with the sampling step between the sampling points satisfying the relevant sampling criterion [1] - [2]. For the typical cases of planar, cylindrical, or spherical scanning, each of the two surface coordinates will correspond to either a straight line with the probe signal along this thus being spatially non-periodic or a full circle with the probe signal along this thus being periodic.

In order to achieve an acceptable signal-to-noise ratio (SNR) in practical near-field antenna measurements, probe signal averaging is employed whether network analyzers [3] or separate signal generators and receivers [4] are employed. That is, the nominal sample of the probe signal at the nominal sampling point is actually an average of multiple, between two and several millions, actual samples. Ideally, signal averaging improves the signal-to-noise power ratio by a factor equal to the number of actual samples per nominal sample [5].

Many measurement systems employ on-the-fly sampling; i.e., the probe is in relative motion with respect to the AUT as probe signal samples are taken. Hence, the multiple actual samples per nominal sample are not taken at the same nominal sampling point but are distributed over a sampling interval; and though this sampling interval must obviously be smaller than the sampling step between the nominal sampling points, the probe signal is not necessarily constant over the sampling interval but may change. In signal processing terms, the averaging is not coherent but incoherent [5] and the nominal sample, resulting from the averaging of the actual samples, does not constitute a point quantity. If, in the post-processing to determine the plane/cylindrical/spherical wave expansion of the AUT field, the nominal probe signal sampling is treated as a point quantity, an error is thus introduced that may give rise to smearing and/or shifting of the radiation pattern.

In practice, the sampling interval is often small compared to the sampling step; hence, the error due the signal averaging with on-the-fly sampling may be very small and even negligible for the measurement uncertainty estimate.

Nevertheless, it is possible to exactly quantify the effect of the signal averaging with on-the-fly sampling, however small this effect is. It is also possible to exactly correct for this effect and recover the nominal probe signal sample at the nominal sampling point with signal averaging but without on-the-fly sampling – for non-periodic as well as periodic probe signals. This correction may be practically relevant when a very high SNR is required thus necessitating a sampling interval comparable to the sampling step.

This work shows that the effect of signal averaging with on-the-fly sampling can be exactly quantified and exactly corrected for. Analytic expressions for the exact signal spectrum in terms of the nominal samples are established and numerical examples based on experimental measurements are presented. Section II reviews the classical case with pulse-sampling of a continuous signal. Section III presents the case of a periodic probe signal; including validation tests based on experimentally measured data. The case of a non-periodic probe signal is analogous and omitted due to the page limitation; but it will be presented at conference. Section IV gives conclusions.

II. PULSE-SAMPLING OF CONTINUOUS SIGNAL

Before investigating the discrete case with pointwise probe signal samples in near-field antenna measurements in Sections III, it is instructive to recall the classical – and perhaps more simple – continuous case.

The continuous sample $s(u)$ of a continuous signal $f(u)$ may be modelled as the convolution of the signal with a rectangular pulse $p(u)$, see Fig. 1. That is,

$$s(u) = \int_{-\infty}^{\infty} f(u')p(u'-u)du' \tag{1}$$

with the pulse of width P and magnitude $1/P$

$$p(u) = \begin{cases} 1/P & \text{for } |u| \le P/2 \\ 0 & \text{for } |u| > P/2 \end{cases} . \tag{2}$$

The spectrum $S(k)$ of the sampled signal $s(u)$ is thus the product of the spectrum $F(k)$ of the signal $f(u)$ and the spectrum $P(k)$ of the pulse $p(u)$; with the latter being a sinc-function. Hence, with k denoting the spectral variable,

$$\begin{aligned} S(k) &\equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} s(u)e^{-jku}du \\ &= F(k)P(k) = F(k)\text{sinc}(kP/2). \end{aligned} \tag{3}$$

The signal $f(u)$ can thus be recovered from the sampled signal $s(u)$ as follows: First, the Fourier transform $S(k)$ is calculated from the sampled $s(u)$. Second, $F(k)$ is calculated by dividing $S(k)$ with $P(k)$; this requires that $P < 2\pi/\max(k)$ to avoid creating artificial singularities with division by zero. Third, the signal $f(u)$ is calculated through the inverse Fourier transform

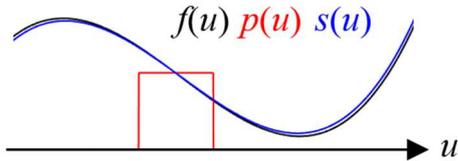$$f(u) \equiv \int_{-\infty}^{\infty} F(k)e^{jku}dk . \tag{4}$$

Fig. 1. Continuous sampling $s(u)$ of continuous signal $f(u)$ with pulse function $p(u)$. Due to the non-zero pulse width, the sampled signal $s(u)$ deviates from the signal $f(u)$ itself; but $f(u)$ can be recovered from $s(u)$ as outlined above.

III. PERIODIC PROBE SIGNAL

This Section III first establishes the analytic relationship for the correct signal spectrum in terms of the nominal samples in Sub-Sections A-C and then presents a series of numerical-experimental examples in Sub-Sections D and E.

A. Analytic Relationships I

A 2π-periodic and spatially bandlimited probe signal w, being a function of the scan coordinate u, can be expressed as a Fourier series with truncation index N,

$$w(u) = \sum_{-N}^{N} c_n e^{jnu} , \tag{5}$$

with the Fourier series coefficients c_n given by

$$c_n = \frac{1}{2\pi} \int_{0}^{2\pi} w(u)e^{-jnu}du . \tag{6}$$

For this type of signal, these coefficients can be determined *exactly* from pointwise signal samples using the finite left-Riemann sum,

$$c_n = \frac{1}{2\pi} \sum_{m=1}^{M} w((m-1)\Delta u)e^{-jn(m-1)\Delta u}\Delta u, \quad \Delta u = \frac{2\pi}{M} , \tag{7}$$

where Δu is the sampling step and M is the number of samples satisfying the sampling criterion $M \ge 2N+1$.

Using signal averaging with on-the-fly sampling, it is not the actual signal $w((m-1)\Delta u)$ that is sampled at the nominal sampling point. Instead, it is a nominal sample $w_{nom}((m-1)\Delta u)$ that is itself an average of multiple actual samples, see Fig. 2; this can thus be expressed as

$$w_{nom}((m-1)\Delta u) = \frac{1}{L} \sum_{l=1}^{L} w((m-1)\Delta u + (l-1)\delta u), \tag{8}$$

where L is the number of actual samples per nominal sample and δu the sampling step between the actual sampling points. The sampling interval $L\delta u$ must obviously be smaller than the sampling step Δu.
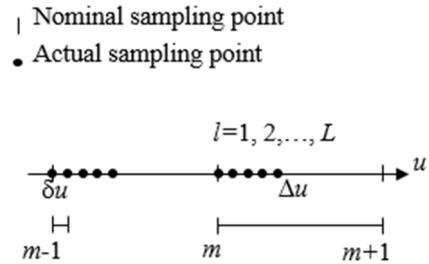

Fig. 2. For signal averaging with on-the-fly sampling, the nominal samples at the nominal sampling points Δu apart are averages of multiple actual samples at actual sampling points δu apart.

Insertion of the nominal sample (8) into the left-Riemann sum (7) does not yield the correct coefficient c_n but an un-corrected coefficient $c_{n,uc}$ that has however a simple relationship with c_n through a correction factor C_n

$$c_{n,uc} = c_n e^{jn(L-1)\delta u/2} \frac{\sin(nL\delta u/2)}{L\sin(n\delta u/2)} \tag{9a}$$

$$c_n = c_{n,uc}C_n, \quad C_n \equiv e^{-jn(L-1)\delta u/2} \frac{L\sin(n\delta u/2)}{\sin(nL\delta u/2)} . \tag{9b}$$

If no signal averaging is used and the nominal sample is thus equal to the actual sample at the nominal sampling point, i.e., $L = 1$, the correction factor $C_n = 1$ and the correct coefficient c_n is obtained directly from the left-Riemann sum. Also, if on-the-fly sampling is not used, i.e., $\delta u = 0$, and all actual samples are taken at the nominal sampling point, again the correction factor $C_n = 1$. However, if signal averaging is employed, i.e., $L > 1$, with on-the-fly sampling, i.e., $\delta u \neq 0$, an un-corrected coefficient results – and when this is employed in (5), an un-corrected signal w_{uc} results which is different from the correct signal w,

$$w_{uc}(u) = \sum_{-N}^{N} c_{n,uc} e^{jnu} . \tag{10}$$

Given a reference signal w, the erroneous effect of signal averaging with on-the-fly sampling can thus be exactly quantified: Using the measured nominal samples w_{mod} in the left-Riemann sum gives the un-corrected coefficients $c_{n,uc}$ which upon insertion into (10) gives the un-corrected signal w_{uc} that can be compared with the reference signal w.

Furthermore, the erroneous effect of signal averaging with on-the-fly sampling can also be exactly corrected for: Using the measured nominal samples w_{mod} in the left-Riemann sum gives the un-corrected coefficients $c_{n,uc}$; upon multiplication with the correction factor C_n (9b) the correct coefficients c_n are obtained, and the corrected signal thus recovered with (5). It is noted that due to the sampling criterion, and since the sampling interval is smaller than the sampling step, the correction factor is non-singular for all n.

B. Analytic Relationships – In Terms of System Parameters

In practice, one does not operate with the quantities L, the number of actual samples per nominal sample, or δu, the sampling step between the actual sampling points. The practical measurement system parameters are the positioner scan speed v_{scan} – set by the operator, the sampling rate for the nominal samples r_{nom} – also set by the operator, and the sampling rate for the actual samples r_{act} – typically fixed by the system. It is seen that the number of actual samples

$$L = r_{act} / r_{nom} , \tag{11}$$

and the step between actual samples

$$\delta u = v_{scan} / r_{act} , \tag{12}$$

so that the sampling interval is

$$L\delta u = v_{scan} / r_{nom} . \tag{13}$$

In terms of these practical measurement system parameters, the correction factor C_n can thus be expressed as

$$C_n = e^{-j\frac{n}{2}\frac{v_{scan}(r_{act}-r_{nom})}{r_{act}r_{nom}}} \frac{r_{act}}{r_{nom}} \frac{\sin(nv_{scan} / 2r_{act})}{\sin(nv_{scan} / 2r_{nom})} . \tag{14}$$

It is noted, that the concept of actual sample employed here essentially means the sample for the highest nominal sampling rate that can be chosen by the operator – in which case the nominal sample equals the actual sample. It may well be that this actual sample is itself an average of multiple underlying samples, but this is thus intrinsic to the measurement system beyond the control of the operator. It is also noted that what is meant by a sample herein is a sample of the complex phasor for the time-harmonic probe signal; such a sample requires multiple A/D conversions of the time-dependent signal which typically has an intermediate frequency such as 45 MHz; again, this is intrinsic to the measurement system beyond the control of the operator.

C. Analytic Relationships – Interpolation Formula

If no signal averaging is used and the nominal sample is thus equal to the actual sample at the nominal sampling point, insertion of (7) into (5) yields the standard interpolation formula for periodic signals expressing the signal at any scan coordinate directly in terms of the actual samples at the nominal sampling points, i.e.,

$$w(u) = \frac{1}{M} \sum_{m=1}^{M} w((m-1)\Delta u) \frac{\sin[(2N+1)U]}{\sin[U]} , \tag{15a}$$

$$U = \frac{u-(m-1)\Delta u}{2} . \tag{15b}$$

If signal averaging with on-the-fly sampling is used, the interpolation formulation would follow by substituting into (5) the corrected coefficient from (9b) wherein the un-corrected coefficient $c_{n,uc}$ is given by (7) with the nominal sample w_{nom} in place of the actual sample w. This would express the correct signal at any scan coordinate directly in terms of the nominal samples at the nominal sampling points. However, in this case it does not seem possible to reduce the double-sum expression to a single-sum expression like (15).

D. Numerical/Experimental Examples – Without Noise

The correction factor C_n (14) relates the correct Fourier coefficients c_n to the un-corrected Fourier coefficients $c_{n,uc}$ due to the signal averaging with on-the-fly sampling. It is seen that $C_0 = 1$ whatever the scan speed or sampling rates; this is due to the fact that $n = 0$ corresponds to the constant harmonic for which there is no difference between taking multiple samples at the nominal sampling point or over the sampling interval. With decreasing/increasing n, the corresponding harmonics vary increasingly over the sampling interval, and the correction factor thus becomes increasingly different from 1. For the highest harmonic $n = N \approx \pi / \Delta u$ and in the extreme case where the sampling interval approaches the sampling step, i.e., $L\delta u = v_{scan}/r_{nom} \approx \Delta u$, $C_N \approx -j\pi / 2$ which is notable.

We now consider examples based on an experimental measurement [6]. Fig. 3 shows a typical probe signal for the E-plane of a SA 12-5.8 standard gain horn operating at 8.35 GHz for a sampling step $\Delta u = 2°$ corresponding to $N = 89$.

This probe signal will be reference for a numerical investigation employing a typical scan speed $v_{scan} = 3°/s$ and a realistic actual sampling rate $r_{act} = 4$ MHz [4]. Thus, the nominal sampling rate r_{nom} must exceed 1.5 Hz to ensure that the sampling interval is smaller than the sampling step.
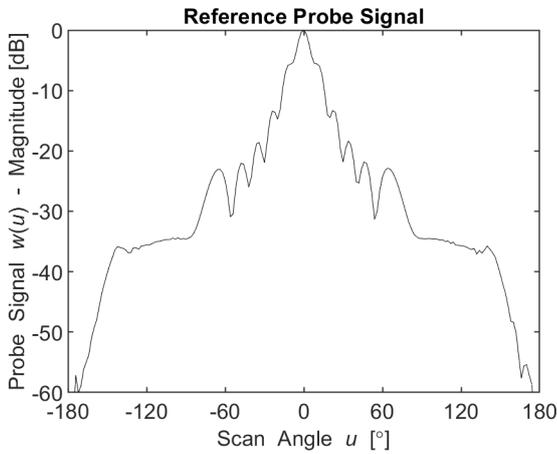
For a typical nominal sampling rate $r_{nom} = 147$ Hz, there are some 27.000 actual samples per nominal sample and the SNR increases ideally by some 44 dB. The sampling interval is around 0.02° which is only 1% of the nominal sampling step with ample range for multiplexing more beams/channels and/or frequencies during the same scan. In this case, the effect of signal averaging with on-the-fly sampling is too small to be seen in the 60 dB dynamic range of Fig. 3. Instead, Fig. 4 shows two zoom-ins comparing the reference pattern with un-corrected and corrected patterns according to the relationships in Sub-Section II.A. It is seen that the deviations of the un-corrected probe signal (blue curve) from the reference signal (black curve) are indeed very small of the order of 0.001 dB to 0.01dB; calculated over all nominal sampling points, the mean of the absolute deviation is around 0.01 dB and the maximum around 0.15 dB. It is also seen that these small deviations can be exactly corrected for; the corrected probe signal (dashed red curve) is identical to the reference probe signal.

Fig. 3. Reference probe signal: E-plane of a SA 12-5.8 standard gain horn operating at 8.35 GHz for a sampling step $\Delta u = 2°$.
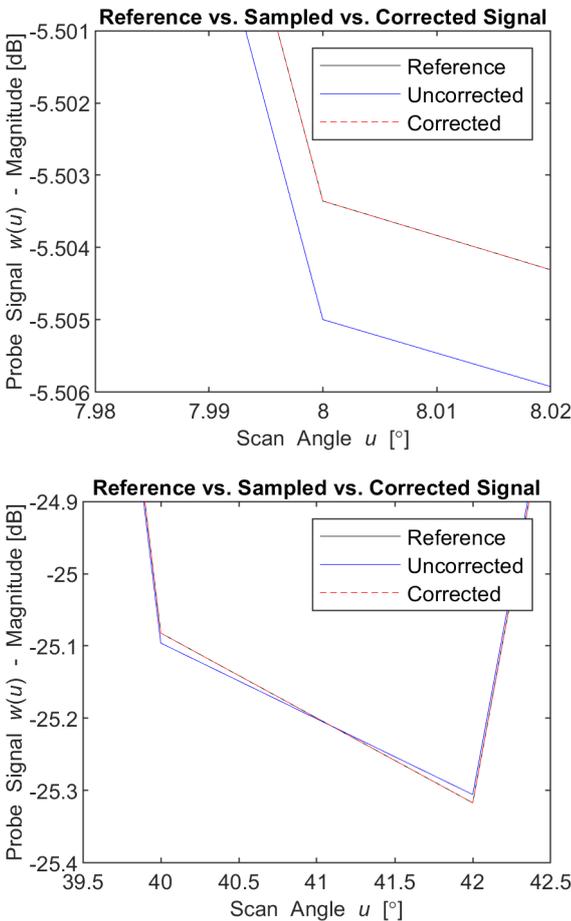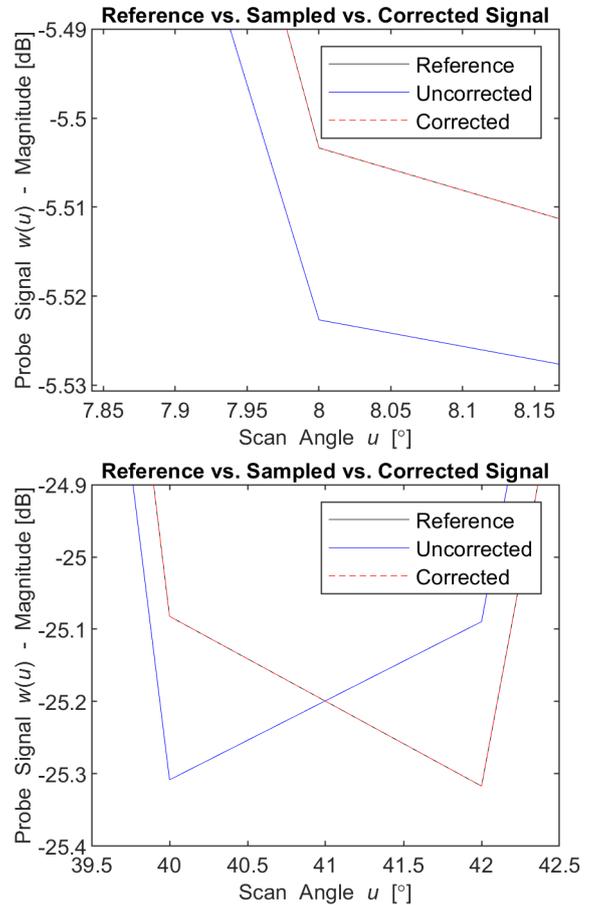

Fig. 4. Comparison of reference signal with un-corrected and corrected signals from signal averaging with on-the-fly sampling for a nominal sampling rate of 147 Hz – for two angular ranges.

Fig. 5. Comparison of reference signal with un-corrected and corrected signals from signal averaging with on-the-fly sampling for a nominal sampling rate of 8 Hz – for two angular ranges.

For a low nominal sampling rate of 8 Hz, there are some 500.000 actual samples per nominal sample and the SNR increases ideally by some 57 dB. The sampling interval is

0.375° which is about 20% of the sampling step. In this case, the effect is just visible over a 60 dB dynamic range; but the two zoom-ins in Fig. 5 show better the deviations which are now of the order of 0.01dB to 0.1dB; calculated over all nominal sampling points, the mean of the absolute deviation is around 0.2 dB and the maximum around 4 dB (though at a low level of about -60 dB). Hence, the deviations are now comparable to other typical error levels in an uncertainty estimate. Again, it is seen that the deviations can be exactly corrected for; the corrected probe signal is identical to the reference probe signal.

E. Numerical/Experimental Examples – With Noise

In practice, measurements are affected by noise and the above correction of the nominal samples based on signal averaging with on-the-fly sampling will not exactly recover the reference signal. In order to quantitatively investigate the effect of measurement noise, the numerical simulation was extended by adding to each actual sample a noise with magnitude randomly generated from a Gaussian probability function with zero mean and variance equal to a specified SNR, and with the phase randomly generated from a uniform probability function. Fig. 6 shows the noise-free reference signal as well as examples of the signals obtained without signal averaging for SNR = 60 dB, 40 dB, and 20 dB, respectively. It is obvious that signal averaging is imperative.
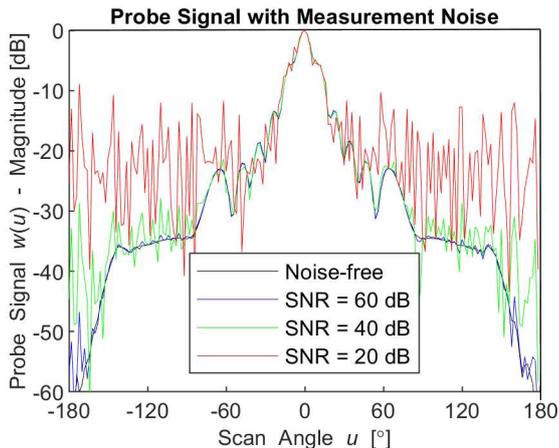

Fig. 6. Probe signals in case of measurement noise (Gaussian magnitude and uniform phase) for different SNR values. The noise-free signal is the same as that of Fig. 3.

Table I lists the average difference between the corrected signal and the reference signal for 12 combinations of 3 nominal sampling rates and 4 SNRs including the noise-free case; the difference is calculated between normalized signals in logarithmic scale. For the 3 nominal sampling rates r_{nom} of 8 Hz, 147 Hz, and 1000 Hz, the number of averaged actual samples per nominal sample is 500.000, 27.000, and 4.000, respectively, giving ideal SNR improvements of 57 dB, 44 dB, and 36 dB, respectively. It is seen that the difference is indeed affected by noise but also that it is in all cases very

small. That is, the correction for signal averaging with on-the-fly sampling performs well also in the presence of measurement noise.

TABLE I. DIFFERENCE BETWEEN REFERENCE SIGNAL AND CORRECTED SIGNAL IN PRESENCE OF MEASUREMENT NOISE

Difference [dB]		SNR [dB]			
		∞	60	40	20
r_{nom} [Hz]	8	10^{-10}	10^{-6}	10^{-6}	10^{-4}
	147	10^{-11}	10^{-5}	10^{-4}	10^{-3}
	1000	10^{-12}	10^{-4}	10^{-3}	10^{-2}

IV. CONCLUSIONS

Signal averaging with on-the-fly sampling introduces a measurement error. Though this error may in many cases be very small and though it can be reduced by choosing a lower scan speed or higher sampling rate, it is of importance to be able properly assessing the error and correcting for it.

The results presented in this paper – the spectral correction factor (14) – make it possible to exactly quantify the error as well as exactly correcting it. As demonstrated, the correction performs very well also in the presence of notable measurement noise. An analogous correction exists for the case of a non-periodic probe signal.

Hence, it is possible not only to properly assess the significance of the error caused by signal averaging with on-the-fly sampling but also – instead of increasing measurement time by lowering scan speed or compromising accuracy by increasing nominal sampling rate – to recover the correct signal however large the sampling interval is relative to the sampling step. This may be of practical relevance for high-accuracy antenna measurements. Though it has been near-field antenna measurements considered herein, the results obviously apply also for compact-range or far-field antenna measurements.

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