Quantum-Assisted Combinatorial Optimization of Reconfigurable Intelligent Surfaces

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***Abstract*—Recently, we have seen an extensive and growing interest in leveraging reconfigurable intelligent surfaces towards smart radio environments. One key question arises on how to efficiently select the phase configuration that produces a desired reflective wavefront. In this work, we proposed a physics-based optimization approach inspired by the quantum mechanical physics of correlated spins. The new idea is grounded on the isomorphism between the electromagnetic scattered power and Ising Hamiltonian. Thereby, the problem of optimizing phase configuration is converted into finding the ground state of the target Ising Hamiltonian. Under this framework, we successfully demonstrated the feasibility of combinatorial optimization for weighted beamforming and diffusive scattering applications.**

***Index Terms*—diffusive scattering, Ising model, reconfigurable intelligent surface, weighted beamforming**

I. Introduction

Reconfigurable intelligent surfaces (RISs) are software-controlled large engineered surfaces with many low-cost passive reflecting elements, where the desired reflective wavefront may be achieved by tuning the local reflection phase and/or amplitude of individual elements [1]. Recently, we have seen a growing interest for leveraging RISs in dynamic manipulation of the wireless propagation environment [2]. The goal is to turn the wireless channel into a smart/reconfigurable space that provides enhanced coverage with high energy efficiency and supporting ultra-fast and seamless connectivity. To harness the full potential of RIS-enabled smart electromagnetic (EM) environment, one needs to rapidly optimize the states of RIS with prescribed objective functions that incorporate specific EM functionalities, e.g., beamforming, focusing, diffusion. This constitutes a substantial computational task due to the enormous number of available degrees of freedom (DOFs).

Our vision is to fuse statistical EM models with quantum computing (QC) algorithms for the rapid optimization of RIS-aided smart radio environment. As an example, the Ising model is widely used in statistical mechanics to describe the spin state of arrays of quantum particles. In [3], we dwell on this analogy and develop an Ising model for the RISs with beamforming/nullforming applications. The EM wave energy is used to calculate the Ising Hamiltonian. The global solution of the problem, i.e., the values of the local reflection phases across the RIS, is obtained by computing the ground state of a biased effective Ising Hamiltonian via quantum annealing.

In this work, we go beyond the rudimentary beamforming applications and demonstrate the feasibility of the proposed methodology for weighted beamshaping and diffusive scattering applications. The emphasis is placed on the construction of quantum-suitable RIS models, which recast the EM scattering problem with multi-element RIS devices into a physical formulation that can be tackled efficiently with novel QC algorithms. The results are believed to be important in the design of large-scale smart EM environments and new wireless infrastructures, including high capacity networks with reduced emission levels, smart skins for EM wave signal processing, and directed energy countermeasures to high power EM coupling.

II. Ising Model for RIS Beamforming

The RIS is typically formed by an array of independently reconfigurable unit cells. In practical RIS structures, one can tune the reflection phase of each RIS unit cells from a finite set of phase states, e.g., the unit cell of 1-bit RIS can be tuned with reflection phase of 0^o or 180^o, and the unit cell of a 2-bit RIS element exhibits four reflection phase states. As such, we can view the RIS optimization as an integer programming model, which searches for an optimal solution over all the combinatorial states of RIS elements. However, this problem has a complexity that scales exponentially with the number of unit cells N (i.e. $O(2^N)$ for 1-bit RIS and $O(4^N)$ for 2-bit RIS). Thus, classical optimization approaches do not scale well with large N, which may create high latency that is unbearable in communication oriented applications.

Alternatively, we can reframe the combinatorial explosion problem into a statistical wave model that is easier to solve with an appropriate quantum computing algorithm. To briefly demonstrate the method, we consider an M-element linear RIS array with the application of desired signal maximization towards the elevation angle of θ^s and azimuthal angle of ϕ^s. The scattered EM field can be written in the Dirac notation as: $|\mathbf{E}\rangle = \sum_{m=1}^{M} \mathbf{G}_m(\theta, \phi) \, |s_m\rangle$, where the RIS basis state s_m represents the element phase modulation, e.g. ± 1 corresponding to the $0/\pi$ phase response, and the $\mathbf{G}_m(\theta, \phi)$ is the element-wise scattering vector calculated with the free-space Green's function. We can then express the EM scattered power as a quadratic model:

$$P(\theta,\phi)=\langle\mathbf{E}|\mathbf{E}\rangle=\sum_{m=1}^{M}\sum_{n=1}^{M}\langle s_n|\,\mathbf{G}_n^*(\theta,\phi)\mathbf{G}_m(\theta,\phi)\,|s_m\rangle \quad (1)$$

From this, we can construct an energy maximization Hamiltonian with an order 2 polynomial. By using symmetry in the scattering vector, the effective Hamiltonian can be constructed as an Ising spin-glass model:

$$H(\theta^s,\phi^s)=-P(\theta^s,\phi^s)=\sum_{m=1}^{M}\sum_{n=m+1}^{M} s_m s_n \bar{J}_{mn}(\theta^s,\phi^s) \quad (2)$$

in which the desired scattering direction is denoted by θ^s and ϕ^s. The computation of spin-spin interaction strength, $\bar{J}_{mn}$, is detailed in [3] and skipped here for brevity.

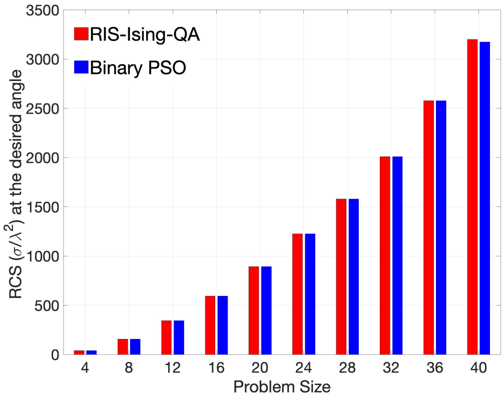
We now convert the problem of achieving the desired signal maximization to finding the ground-state solution of Ising Hamiltonian for the M spin system in (2). To find the the ground-state solution effectively, one appealing way forward is to leverage recent advances in the adiabatic QC hardware, so-called quantum annealer (QA) [4], which received considerably interests lately due to the number of available qubits and programmability [5]–[9].

The particular physical QA hardware considered in this work is the D-Wave Advantage system [10], which has 5,000 functional quantum bits (qubits) represented by circulating currents in superconducting loops. The Ising model is compiled into the D-Wave QA device through a process of embedding and de-embedding. The corresponding quantum Hamiltonian with Ising spins in a transverse field is given by:

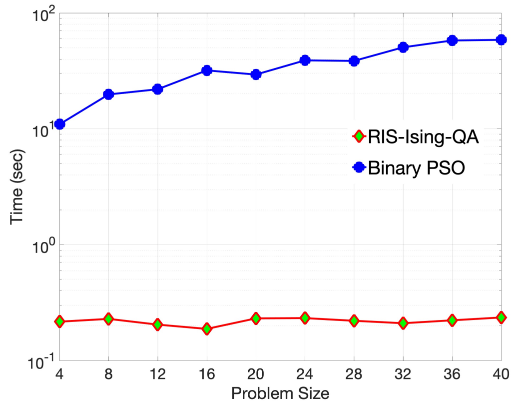
$$\mathcal{H}(t)=-\mathcal{A}(t)\left(\sum_{m=1}^{M}\hat{\sigma}_m^{\mathrm{x}}\right)+\mathcal{B}(t)\left(\sum_{m=1}^{M}\sum_{n=m+1}^{M}\hat{\sigma}_m^{\mathrm{z}}\hat{\sigma}_n^{\mathrm{z}}\bar{J}_{mn}\right) \quad (3)$$

where $\hat{\sigma}_m^{\mathrm{x,z}}$ are the Pauli spin matrices. The $\hat{\sigma}_m^{\mathrm{z}}$ represent the spin projections along either the $+\mathrm{z}$ or $-\mathrm{z}$ direction (taking values +1 for spin up and -1 for spin down). The $\mathcal{A}(t)$ stands for the transverse Hamiltonian due to an applied transverse field in the x-direction. The quantum annealing process starts at time $t=0$ with $\mathcal{A}(0)\gg\mathcal{B}(0)$. The system is then evolved by decreasing $\mathcal{A}(t)$ and increasing $\mathcal{B}(t)$ until the annealing time t_f is reached. If the increase in $\mathcal{B}$ is slow enough, the adiabatic theorem ensures that the final state of the system is the ground state of the target Hamiltonian. Namely, the qubits have dephased to classical systems, and the $\hat{\sigma}_m^{\mathrm{z}}$ can be replaced by classical spin variables, $\hat{s}_m$, which indicates the optimal configuration of RIS.

The first numerical experiment in this paper is to compare the performance of proposed work with classical metaheuristic optimization algorithms. Consider a TE polarized plane wave normally incident upon the 1-bit RIS array. We aim to optimize the RIS phase profile, such that the scattered power is maximized at a desired direction $\theta^s=15^o$ and $\phi^s=90^o$. The obtained radar cross section (RCS) values at the desired angle using the proposed RIS-Ising-QA and the binary particle swarm optimization (PSO) approach [11] are shown in Fig. 1. We observe that, even though the qualities of the optimized results are similar, the proposed RIS-Ising-QA algorithm is approximately 100 times faster than the PSO optimizer. This encouraging result motivates the investigation of the RIS-Ising model for more advanced wireless and EM applications.



(a) Optimized RCS value at the desired angle



(b) Time-to-solution

Fig. 1. Numerical experiment for RIS beamforming at $\theta^s=15^o$ and $\phi^s=90^o$. The time-to-solution includes CPU execution time for computing Ising coefficients and QPU access time for collecting energy samples of 10 runs.

III. Weighted Multiple Beamforming

A. Motivation for Weighted Beamforming

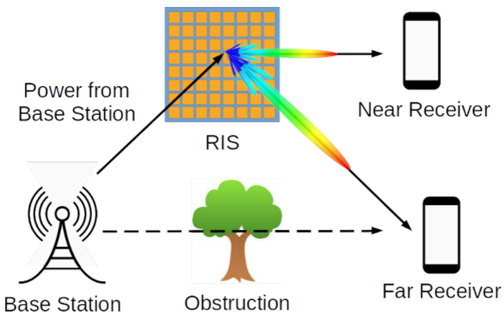


Fig. 2. RIS-aided NOMA wireless network. The RIS can help to redirect the signal to receivers blocked by obstruction in line-of-sight with the base station. The RIS should also be able to balance the beam power towards mobile receivers at different distances.

One important application of RIS in wireless communication is to create a virtual line-of-sight link between transmitter (Tx) and receiver (Rx) when the direct-link between them is blocked by an obstacle, as shown in Fig. 2. In a non-orthogonal multiple access (NOMA) network, there are multiple mobile receivers that simultaneously require a considerable amount of signal energy from the transmitting base station. To achieve high energy efficiency, we want the RIS to redirect signals from a single base station to distribute receivers with the appropriate signal power. In particular, the RIS may be designed to perform beamforming towards nearer receivers with lower signal power and beamforming towards farther receivers with higher signal power. Consequently, all mobile receivers can meet the minimum signal-to-noise ratio requirement.

B. Construction of Ising Hamiltonian

Given the problem statement in Fig. 2, an extension of (2) can be formulated as a fourth-order Ising Hamiltonian with enhanced beamforming flexibility. Assuming desired scattering directions towards two Rxs are θ_1^s, ϕ_1^s, and θ_2^s, ϕ_2^s, we have:

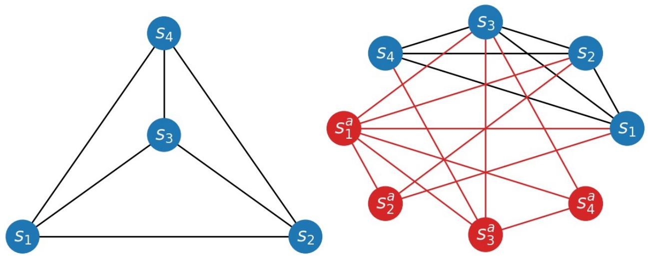
$$\begin{aligned} H\left(\theta_1^s, \phi_1^s, \theta_2^s, \phi_2^s\right) = \big[& -P^2\left(\theta_1^s, \phi_1^s\right) - P^2\left(\theta_2^s, \phi_2^s\right) \\ & + \Lambda \left(P\left(\theta_1^s, \phi_1^s\right) - W * P\left(\theta_2^s, \phi_2^s\right)\right)^2 \big] \end{aligned} \tag{4}$$

where the first two terms in the right hand side ensure the scattered beams are directed toward the two Rxs. The last term controls the scattering power ratio between the two beams, in which Λ is the penalty strength and W is the weighting factor.

We note that the product of scattered powers leads to the fourth order polynomial in the Ising model, also known as four-body interaction among four Ising variables $s_m s_n s_i s_v$. D-Wave QA hardwares are not capable of solving these terms directly, as current annealers only allow polynomials with a maximum order of 2. Thus, we need convert the fourth order terms to second order ones through a process of quadratization.

Specifically, we have made use of a polynomial penalty approach, similar to methods introduced in [12], to reduce higher order terms to a sum of pairwise-interaction terms. To briefly show the algorithm, we consider a three-body interaction term of $s_1 s_2 s_3$, where s_i is the i^{th} Ising variable. By introducing 2 auxiliary variables (i.e. s_1^a and s_2^a) and a weighted quadratic penalty function, the triplet interaction term, $s_1 s_2 s_3$, can be represented as: $s_1^a s_1 + V(-\frac{1}{2}s_1^a - \frac{1}{2}s_2 - \frac{1}{2}s_3 - s_2^a + \frac{1}{2}s_2 s_3 + (s_2 + s_3)s_2^a + \frac{1}{2}(s_2 + s_3)s_1^a + s_1^a s_2^a)$, where V is the weighting factor for the penalty function. This method can be further extended to the quadratization of 4-body interaction term by first converting the 4-body term to 3-body terms and then 3-body terms to 2-body interaction terms, recursively. As a result, four new auxiliary variables are added for each 4-body interaction term in the Hamiltonian.

Fig. 3(b) graphically illustrates the resulting Ising model for two beamforming application as compared to the case of single beamforming in Fig. 3(a). In the graph, the blue nodes represent the original Ising variables related to the phase of each RIS cell and the red nodes represent the auxiliary variables introduced from the aforementioned quadratization procedure. The connected edges describe the interactions between these variables. The benefit of including the additional terms allows us to simulate the higher order optimization problem using current quantum annealers.



(a) Single beamforming (b) Two beamforming

Fig. 3. Graph of spin-spin interaction between original Ising variables (blue nodes) and auxiliary variables (red nodes) of a 2 by 2 RIS for (a) single beamforming case (no auxiliary variables) and (b) multiple beamforming case.

C. Numerical Experiment

In this subsection, a 5 by 5 RIS array with element size d=1λ and normally incident plane wave is used to demonstrate the RIS-Ising annealing optimization for weighted multiple beamforming. We start with the equal beamforming study. Two angular directions of interest are defined to direct the scattering energy. We set the first beam towards $\theta_1^s = 15^o, \phi_1^s = 110^o$, whereas the second beam is directed towards $\theta_2^s = 15^o, \phi_2^s = 45^o$. Moreover, the Λ and W in (4) are set to 1 for equal weightings for both beams. By minimizing the Ising Hamiltonian, we can achieve good beamforming in the specified angular directions with approximately equal weightings, as depicted in Fig. 4. Furthermore, the ratio of scattering intensity towards $\phi_1^s = 110^o$ is approximately 1.1 times the scattering intensity towards $\phi_2^s = 45^o$. Thus, this shows a minimal energy bias between the two scattered beams.

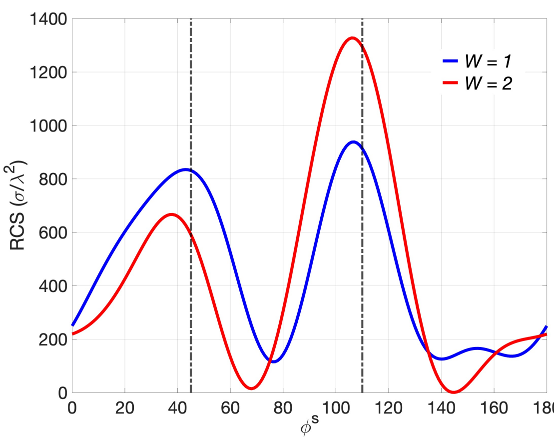


Fig. 4. The radar cross section (RCS) plots at $\theta^s = 15^o$ angular plane obtained from two optimized RIS phase profiles by setting W=1 and W=2.

Next, we shift our focus to the weighted multi-beamforming case. We use the same simulation configuration, except with the weight W changed to 2. By minimizing this Hamiltonian, we aim to create two beams with a weighted bias factor of 2 against the second beam. From Fig. 4, we can observe that the scattering intensity at $\phi_1^s = 110^o$ increases while the scattering intensity at $\phi_2^s = 45^o$ decreases with respect to the original scattering pattern obtained with W = 1. Moreover, the ratio of scattering intensity towards $\phi_1^s = 110^o$ is approximately 2.19 times the scattering intensity towards $\phi_2^s = 45^o$. Therefore, it is concluded that the proposed fourth order Hamiltonian function in (4) enables the optimization of RIS for complex multiple weighted beamforming applications.

IV. Diffuse Scattering

A. Electromagnetic Signature Reduction

Besides beamforming and focusing in communication networks, the RIS can be of great value to other commercial and defense applications. One such example is the use of RIS as an EM stealth solution for military scenarios. The demand for light-weight and low-profile materials for stealth technology in military applications is ubiquitous. Traditionally, absorbing materials are used to lower the EM signature of platforms. However, the performance of these materials usually scales with the material thickness. In cases whereby the material thickness is a hard constraint, the diffusive RIS provides a suitable alternative. By redirecting energy in all directions, the reflected EM energy is dispersed and thus reducing the monostatic EM signature, as illustrated in Fig. 5.

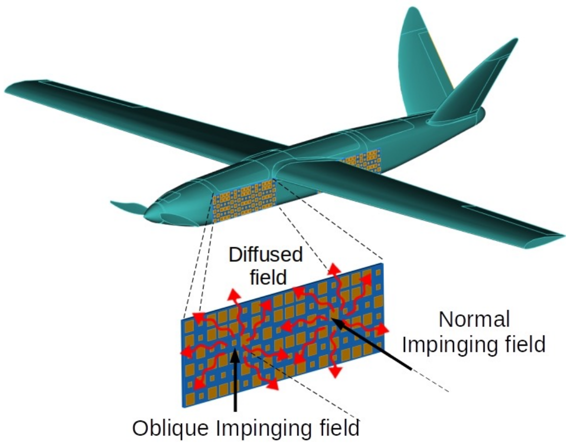


Fig. 5. Diffusive RIS that creates a quasi-uniform diffuse scattering field would reduce the electromagnetic signature of the platform.

In the literature, the scaling laws and bounds on diffusive metasurfaces are discussed in [13]. A sub-optimal generalized Golay-Rudin-Shapiro (GRS) code is utilized to generate the 1-bit coding pattern for the normal plane wave incidence case. As shown in Fig. 5, oblique incident fields may also impinge on the diffusive RIS mounted on the platform. Thus, it is important to assess the diffusive performance in such cases. In this section, we present a RIS-Ising model for diffuse scattering, which incorporates the attributes of element-specific scattering factor and angular-dependent excitation, also can be efficiently solved by quantum annealing algorithms.

B. Construction of Ising Hamiltonian

In the ideal case, a perfect diffusive scatterer would direct energy equally in all directions, resulting in a uniformly low diffused isotropic EM field. In other words, the scattered power would spread throughout the angular domain. Based on this observation, we aim to construct an Ising Hamiltonian with a minimal variance in the scattering intensities.

The conventional way to compute the variance of a scattered power $P(\theta, \phi)$ in the angular domain follows the expected value of the squared derivation from the mean power: $\sigma^2 = \mathrm{E}\left[(P(\theta,\phi) - \bar{P})^2\right]$. However, this method may be impractical as the mean power $\bar{P}$ is inaccessible. Alternatively, we first generate a finite sample of N angular observations, then construct a Hamiltonian associated with the population variance of the scattered power from the RIS:

$$H_{\text{var}} = \frac{1}{N^2} \sum_{i}^{N} \sum_{j>i}^{N} \left[P(\theta_i, \phi_i) - P(\theta_j, \phi_j)\right]^2 \tag{5}$$

whereby N represents the number of scattering angles used to compute the population variance of the scattered power.

From Eq. 5, we can observe that the variance Hamiltonian requires the product of two second-order function $P(\theta, \phi)$. Similar to the weighted multiple beamforming case, we would end up with 4-body interaction terms that are incompatible with current annealing solvers. Thereby, we will adopt the same quadratization procedure to convert these 4-body interaction terms to a sum of 2-body interaction terms recursively. Finally, the obtained quadratic RIS-Ising model is solved by D-Wave hybrid quantum-classical annealing solver.

C. Numerical Experiments for Diffusive Scattering RIS

In this subsection, we first demonstrate the performance of the RIS-Ising model for diffuse scattering under normal incident fields. In the numerical experiment, we employ a two-step optimization strategy similar to the one used in related literature [13], [14]. Specifically, instead of optimizing the entire 2D RIS, we have performed two 1D optimizations along each side of the RIS. Following which, the 1D solutions from these sub-problems are used to derive the 2D RIS configuration via an outer product operation. As compared to the related literature, the proposed work accounts for the polarization of incident field and the element factor of the RIS unit cells through the construction of diffuse RIS-Ising model. Hence, this would encourage better quality solutions.

To benchmark the performance of the proposed approach, we define a figure of merit (FOM) and a physical limit of diffusive RIS for this numerical study. First, the Radar Cross Section (RCS) ratio will be used as the figure of merit [14]. To elaborate, the RCS ratio is the ratio between the highest RCS value from the diffusive RIS to the highest RCS value of a perfect electrical conductor (PEC) plate with the same size. Hence, this FOM describes the performance of the diffusive RIS to suppress high RCS values in all directions. Second, the Isotropic Array Factor (IAF) limit [13] is used as the best possible performance of diffusive RIS. This limit was derived with the assumption of a constant directivity in the array factor. Thus, it may be considered the physical bound of diffusive RIS performance. By using these two definitions, we can evaluate the quality of the optimized solutions.

The example considered here is a M by M rectangular RIS array with binary phase response in the element reflection coefficient. The size of the RIS element as d= 1λ, and the TE polarized plane wave is normally incident upon the array. As is seen from Fig. 6, the RCS ratio obtained from the diffuse RIS-Ising model follows closely to the IAF limit. The results calculated with the RIS configuration using the GRS-P and GRS-Q coding sequences are also included for comparison. The experiment demonstrates that our proposed approach is effective in optimizing diffusive RIS configurations for the normally incident plane wave.

It will also be interesting to apply the optimization approach for the oblique incident angle case. We assume that the incident TE polarized plane wave illuminating from $\theta^i = 75^o, \phi^i = 270^o$. The RCS ratio of the optimized RIS configuration using the diffuse RIS-Ising model are illustrated in Fig. 7. The results using the GRS-P and GRS-Q coding sequences are also included for comparison. The study verifies the flexibility of our Ising Hamiltonian approach in deriving diffusive scattering patterns at oblique incident angles.

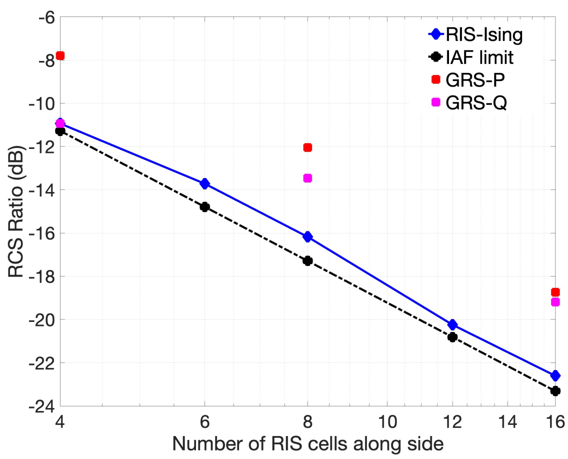


Fig. 6. RCS ratio of diffusive RIS when illuminated at normal incident angle.

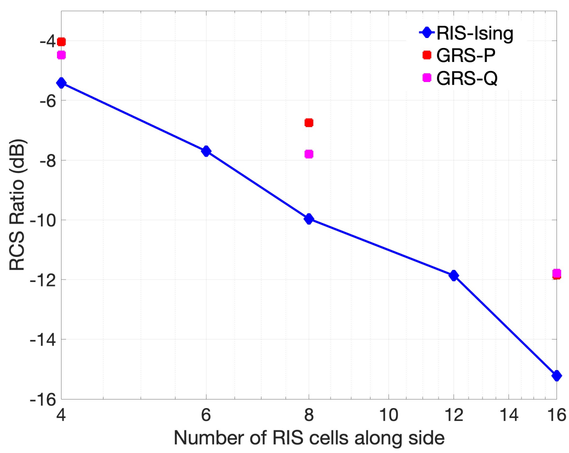


Fig. 7. RCS ratio of the optimized diffusive RIS when illuminated at oblique incident angle ($\theta^s = 75^o$) and ($\phi^s = 270^o$).

V. Conclusion

The RIS is emerging as a key technology for the next-generation of mobile network. The goal is to turn the wireless environment into a smart, reconfigurable space that plays an active role in the communication performance. To harness the full potential of RIS-enabled smart radio environment, we need to rapidly optimize the states of RIS devices with prescribed objective functions.

The proposed work stands on the fusion of statistical mechanics models with quantum computing algorithms to overcome the high computational optimization complexity. The philosophy is to recast the RIS-aided wireless problem into a physical formulation that can be tackled efficiently with emerging QC hardware. This study shades light on the relevance of Ising Hamiltonian for controlling complex EM scattering in real-life scenarios of communication and defense engineering. Future research will be devoted to study the scalability of the optimization approach in solving practical problems involving the deployment of large RIS structures.

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