COMPLEX CONICAL BEAM EXPANSION FOR THE ANALYSIS OF BEAM WAVEGUIDES

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Abstract— **This paper discusses a novel method for computing propagation and reflection of beams in a beam waveguide by combining a new type of wave-objects with Physical Optics. The wave objects are generated starting from the electric field spectrum in the focal plane, by applying the Fourier Series and the Generalised Pencil-of-Function expansion. This transforms the radiation integral to a form which rigorously respects the wave equation and can be evaluated in analytical closed form. The wave objects are launched to the reflector and reflected to the subsequent focal plane via Physical Optics. By computing the Fourier spectrum in the new focal plane it is possible to re-expand the reflected field into a new set of wave-objects, which is the foundation of beam waveguide analysis.**

I. INTRODUCTION

The analysis of reflector systems at mm-wave frequencies requires the development of new analysis methods particularly suited for that frequency and size range. For example, a beam waveguide can be composed of a sequence of reflectors, each of which can be a hundred wavelengths in diameter. Hence, instead of discretising the analysis domain like classical numerical methods do (e.g. Method of Moments or even Physical Optics), the methods developed for this area express the field in terms of a relatively low number of beams. Beams are, generally speaking, wave-objects of higher complexity whose propagation, reflection, and diffraction if possible, can be resolved at least asymptotically in a closed form. The aim is to develop a modular process, whereby the total reflected field can be re-expanded into a new sum of beams, and the whole procedure can be repeated, so as to enable the analysis of complex multi-reflector systems, e.g. the beam waveguide.

A number of different objects have been introduced that are more or less suitable to the exposed analysis approach. These are the Gaussian Beams [1, 2], higher-order Gauss-Laguerre or Gauss-Hermite modes [3, 4], Complex Source Points [5], etc. Recently, we presented a new kind of wave objects (WO's) [6], introducing a formulation that has potential advantages over other approaches in that it satisfies the following important aspects: 1) the WO's respect the wave equation in all space where they are valid, and 2) the generation of these WO's is done in an efficient and natural way starting from the aperture field expansion. Moreover, the WO's possess analytical expressions in both spatial and spectral domain and the zero order WO coincides with an ordinary spherical wave field. All these properties make the WO promising in the description of multi-reflector systems, whose conventional PO analysis results sometimes prohibitively slow.

II. CYLINDRICAL WAVE BEAMS

The WO is defined through its Fourier Transform as

$$W_n\left(\rho,\phi,\tilde{z}\right) = e^{-jn\phi} \int_0^\infty \frac{e^{-j\tilde{z}\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}} J_n\left(\rho k_\rho\right) k_\rho dk_\rho . \quad (1)$$

It has been demonstrated in [6], that (1) can be evaluated in closed form in the space domain by a simple recurrence formula for every index n. For $n = 0$, the representation (1) coincides with the Sommerfeld representation, and the zero-order wave-object is found as $W_0 = j\exp(-jkr)/r$. A similar result is obtained for $n = 1$. Bearing in mind the recurrence relation between the wave-objects, it can be shown that the WO's respect the wave equation for every n.

The WO's are generated automatically from the spectral-domain aperture field representation, which is expressed here in cylindrical coordinates for convenience:

$$I(\rho,\phi,z) = \frac{1}{8\pi^2 j} \int_0^\infty \int_0^{2\pi} g\left(k_\rho,\alpha\right) e^{-j\rho k_\rho \cos(\alpha-\phi)} \frac{e^{-jz\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}} k_\rho dk_\rho d\alpha \quad (2)$$

Here, $g\left(k_\rho,\alpha\right)$ is the spectrum of any of the Cartesian components of the electric (or magnetic) field. The process of beam generation is done in two steps. First, $g\left(k_\rho,\alpha\right)$ is expanded via FFT in a Fourier series in the angular coordinate: $g\left(k_\rho,\alpha\right) = \sum_{n=-\infty}^{\infty} c_n\left(k_\rho\right) e^{-jn\alpha}$; next, the Generalized Pencil of Functions (GPoF) expansion is applied to the Fourier

coefficients: $c_n(k_\rho) = \sum_{m=-M}^{M} d_{mn} e^{b_m \sqrt{k^2 - k_\rho^2}}$. The double integral reduces to this double summation:

$$I = \sum_n c_n \sum_{m=-M}^{M} d_{mn} W_n(\rho, \phi, z + jb_{mn}), \qquad (3)$$

where $W_n(\rho, \phi, \tilde{z})$ is the n-th order wave object, as defined in (1), evaluated at the point $(\rho, \phi, \tilde{z})$ with a complex coordinate $\tilde{z} = z + jb_{mn}$. This means that the initial radiation integral has been reduced to a sum of functions that are well-behaved in the whole upper half-space ($\text{Re}\{\tilde{z}\} > 0$).

III. WAVE OBJECTS VECTORISATION

The quantities defined in (1)-(3) are all scalar quantities. In order to be able to work with electromagnetic fields, it is necessary to vectorise the above procedure. To do that, we have to re-examine eq. (2) and see how it relates to fields and /or potentials. If $g(k_\rho, \alpha)$ is the spectrum of one of the tangential components of the electric (or magnetic) field in the aperture plane, there are two possible approaches to the vectorisation. In the first approach, we can substitute $g(k_\rho, \alpha)$ with $h(k_\rho, \alpha) = 2j\sqrt{k^2 - k_\rho^2}\, g(k_\rho, \alpha)$ in (2), and the relation becomes the classical inverse Fourier-transform, which relates each field component's spectrum at the plane $z = 0$ to the corresponding radiated field at the plane $z = z_0$. Applying the GPOF expansion and the wave-objects in this way can be regarded in a way as the acceleration of the Fourier transformation, because there is no more need for performing the double integration. If, on the other hand, the discrete transformation is used (DFT or FFT), there is another advantage of using the wave-objects, which is that they alleviate limitations on the resolution – once the spectrum is expanded in the GPOF series, the field can be instantly computed in any point of the half-space.

The first approach, however, does not allow computing of the component of the field normal to the aperture. To do that, we resort to the vector wave potentials $\mathbf{A}$ or $\mathbf{F}$. In fact, if $g(k_\rho, \alpha)$ is the spectrum of one electric field component (or the related equivalent magnetic current), the quantity defined by the integral is recognized as the related component of the radiated vector electric potential $\mathbf{F}$. It should be noted that the equivalence theorem allows us to choose whether to use equivalent electric currents, equivalent magnetic currents, or both. If we use only tangential electric field components on the aperture, the potentials relate to the fields [7] as:

$$\mathbf{E} = \hat{x}\left(\frac{1}{\varepsilon}\frac{\partial F_y}{\partial z}\right) - \hat{y}\left(\frac{1}{\varepsilon}\frac{\partial F_x}{\partial z}\right) + \hat{z}\left(\frac{1}{\varepsilon}\left(\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x}\right)\right).$$
$$(4)$$

In this approach it is therefore necessary to compute the derivatives of the wave-objects with respect to all three coordinates, which can be done either numerically or analytically. The use of analytical derivatives of WO's is preferred, because it offers better accuracy and faster operation. However, their derivation is somewhat involved and will be omitted here.

IV. ANALYSIS OF REFLECTOR SYSTEMS USING WAVE-OBJECTS

Fig. 1 shows the basic analysis block of a beam waveguide. For simplicity, let us assume that the reflector under observation be an elliptical reflector. In a well-designed beam waveguide, the beam waist of the incident wave will lie in the same plane as the first focal point of the elliptical reflector, and upon reflection the reflected beam will focus in the plane containing the second focal point. The analysis of a beam waveguide using wave-objects consists of the following basic steps, which can be repeated indefinitely. The incoming field's spectrum is expanded with GPOF into wave-objects in the aperture focal plane. Each beam propagates independently to reflector, and Physical Optics (PO) is used for treating its scattering. The PO field can be recollected either at the second focal plane, or in the far field region. In the first case, the Fourier transformation can be used to arrive at the spectrum of the scattered field, while in the second case the inverse near-field to far-field transformation [8] can be applied for the same purpose, each with its own set of advantages and disadvantages. After we have obtained the outgoing field's spectrum, we can repeat the procedure for every subsequent reflector.
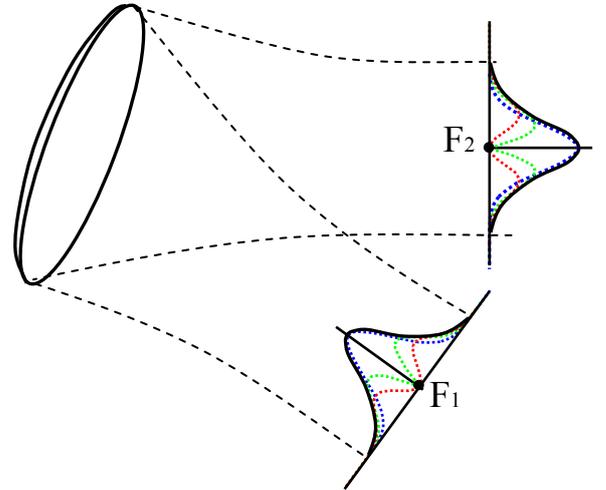

Fig. 1 Basic segment of a Beam Waveguide

The main advantage of this approach is that it requires only a few beams at each step, due to the angular selectivity property of wave-objects, which saves much computational time. In the following sections, we will demonstrate the crucial aspects of the analysis on two examples.

V. NUMERICAL EXAMPLES

The first test demonstrates the validity of the vectorisation strategy. A TM circular waveguide mode distribution is expanded into wave-objects and irradiated, and the values of the radiated field are compared to those obtained via direct spatial-domain radiation integral, which can be found in many textbooks [9]. This particular aperture field distribution has been selected because it possesses closed form expressions in both spectral and spatial domain, therefore enabling us to have an analytical referent solution. It is also worth mentioning that circular waveguide modes are the basic output of a horn mode-matching analysis, and consequently their expansion in wave-objects is of significant practical concern.

A TM_{mn} circular waveguide mode is defined by [7]

$$\mathbf{E}_{mn}\left(\rho,\phi\right) = \alpha_{mn} J_m'\left(\alpha_{mn}\rho\right)\cos\left(m\phi\right)\hat{\rho}$$
$$-\frac{m}{\rho}J_m\left(\alpha_{mn}\rho\right)\sin\left(m\phi\right)\hat{\phi} \quad . \tag{4}$$

Its Fourier transform is

$$\tilde{E}_{x,mn}\left(k_\rho,\alpha\right) = j^{m-1}2\pi\frac{\alpha_{mn}r_w}{\alpha_{mn}^2 - k_\rho^2}\cdot\cos\left(m\alpha\right)\cos\left(\alpha\right)$$
$$\cdot\left\{\alpha_{mn}J_{m+1}'\left(\alpha_{mn}r_w\right)J_{m+1}\left(k_\rho r_w\right) - k_\rho J_{m+1}\left(\alpha_{mn}r_w\right)J_{m+1}'\left(k_\rho r_w\right)\right\}$$

$$\tag{5a}$$

$$\tilde{E}_{y,mn}\left(k_\rho,\alpha\right) = j^{m-1}2\pi\frac{\alpha_{mn}r_w}{\alpha_{mn}^2 - k_\rho^2}\cdot\cos\left(m\alpha\right)\sin\left(\alpha\right)$$
$$\cdot\left\{\alpha_{mn}J_{m+1}'\left(\alpha_{mn}r_w\right)J_{m+1}\left(k_\rho r_w\right) - k_\rho J_{m+1}\left(\alpha_{mn}r_w\right)J_{m+1}'\left(k_\rho r_w\right)\right\}$$
$$\tag{5b}$$

Here, J_m is the Bessel function of m-th order, r_w is the waveguide radius, while $\alpha_{mn} = \chi_{mn}/r_w$, where χ_{mn} is the n-th zero of J_m. The spatial and spectral distributions of the x-component of a TM_{01} mode are shown in Fig. 2. The y-component is the same, only rotated by 90 degrees.
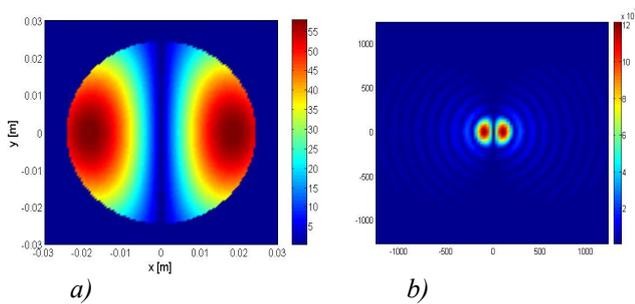

Fig. 2. a) Magnitude of the E_x component of a TM_{01} circular waveguide mode aperture field ($r_w = 0.024049$ m). b) Magnitude of the corresponding spectrum.

Fig. 3 shows the comparison of the directly radiated field of a TM_{01} mode (via spatial radiation integral), and the WO-radiated field, for all three components of the electrical field. For this example, the wave number was $k = 100$ and the magnitude of the radiated mode was unity. The field points were calculated on a x-directed straight line ($y = 5\lambda$) lying in the plane $z = 10\lambda$. The spectrum was sampled in 44 points and represented with 7 GPOF exponents. Only 14 wave-objects were necessary to completely reconstruct the field of the TM_{01} mode. As can be seen in Fig. 3, excellent agreement between direct integration and the wave-objects expansion was achieved, confirming the choice of the vectorisation strategy.
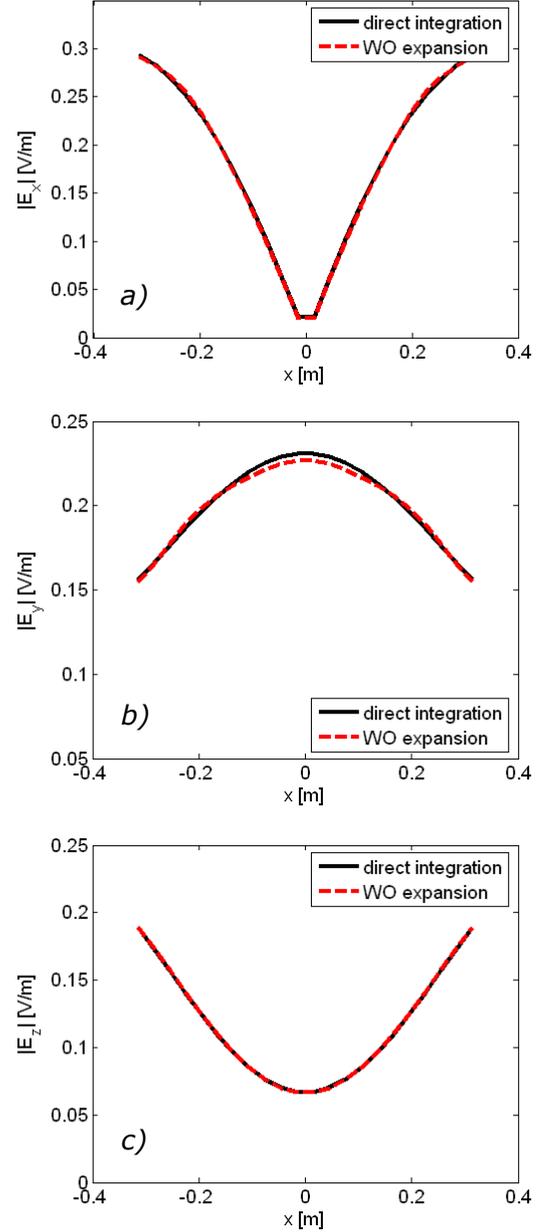

Fig. 3. The radiated field components of a TM01 circular waveguide mode. The waveguide radius is 0.02405 m, and the wavenumber is k = 100. a) Magnitude of Ex ; b) magnitude of Ey; c) Magnitude of Ez.

The second example serves to validate the so-called recollection of the scattered WO-fields and their re-expansion into wave-objects via the described procedure. In this case we considered an elliptical reflector defined as the intersection of an ellipsoid (with semi-axes $a = 8\lambda$, $b = 4\lambda$) with a cone whose nose angle is θ_c (here 30º) and whose apex coincides with one of the two focal points of the ellipsoid, as shown in Fig. 4.
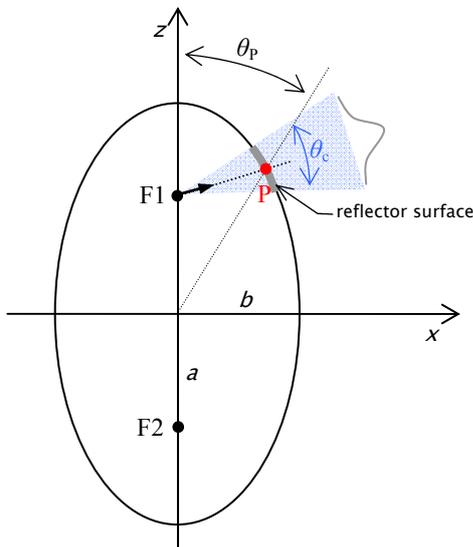

Fig. 4. Geometry of the ellipsoidal reflector. The beam is radiated from focal point F1 towards the point on the ellipsoid for which $\theta_P = 40$º, while the cone nose angle is 30º.
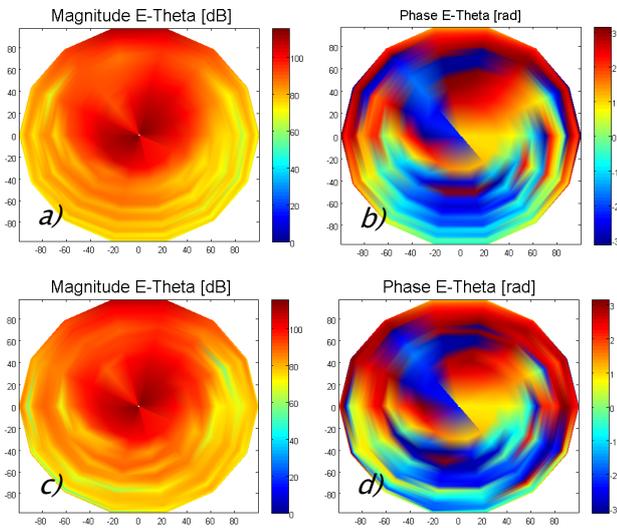

Fig. 5. Magnitude (a) and phase (b) of the PO-radiated field. Magnitude (c) and phase (d) of the same field reconstructed with wave-objects. Both field values are computed on a semi-sphere of radius 100λ whose origin coincides with focal point F2.

The reflector was illuminated by a complex source point located at focal point F1, whose complex displacement gave a Gaussian-type taper to the incident field, with the equivalent Gaussian beam waist $w_0 = 1\lambda$. Note that a complex point source is equivalent to our wave-object of zero order, W_0, as has been demonstrated in [6]. Therefore, our aim was to illuminate the reflector with a wave-object and determine whether or not the reflected field could be re-expanded into wave-objects and reconstructed in the far-field zone. The result was confronted with directly calculated PO fields.

The reflected PO field was computed on a semi-sphere of radius 100λ. The spectral components of the reflected field in the beam waist plane (which contains the second focal point F2) were found through the inverse near-field to far-field transformation. Only 36 wave-objects were employed to reconstruct the radiated fields. The WO field was compared to the original one on a semi-sphere of radius 100λ. The agreement between the results is very good, as shown in Fig. 5.

VI. CONCLUSIONS

In this paper, the complex conical wave objects have been applied to the analysis of a basic building block of a beam waveguide. To be able to analyse a realistic reflector system, it is necessary to vectorise the wave-objects and choose a convenient strategy for recollecting and re-expanding the reflected fields. The proposed method was tested on a case of an elliptical reflector, and the preliminary results show very good agreement with directly calculated PO fields.

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