Small-Scale Variations of Cross-Polar Discrimination in Polarized MIMO Systems

François Quitin #*, Claude Oestges *, François Horlin #, Philippe De Doncker #

#*OPERA Department, Université Libre de Bruxelles (ULB)*
CP 165/81, Av. F.D. Roosevelt 50, 1050 Brussels, Belgium
fquitin@ulb.ac.be

**Microwave Laboratory, Université catholique de Louvain (UCL)*
Maxwell bldg., place du Levant 3, B-1348 Louvain-la-Neuve, Belgium

***Abstract*—Cross-polarized MIMO systems are an attractive way to reduce equipment size while maintaining low inter-antenna correlation. The amount of leakage from one polarization to another is usually represented by the cross-polarization discrimination (XPD), making it a fundamental parameter of cross-polarized antenna systems. Starting from the definition of Rayleigh and Rice fading channels, we show that the small-scale variations of the XPD follow an F-distribution and a doubly non-central F-distribution respectively in Rayleigh and Ricean fading channels. This distribution is compared with the log-normal distribution that is usually reported in literature for the variations of the XPD. Consequences of this distribution are investigated, and it is shown that the variations of the XPD and the Ricean K-factors are linked by a non-trivial relationship. Measurements have been carried out in an indoor environment at 3.6 GHz to validate the theoretical F-model. The theoretical model and the experimental data are compared visually, by means of the mean square error and by means of the Kolmogorov-Smirnov test. The theoretical F-model shows excellent agreement with the measured XPD variations.**

***Index Terms*—MIMO, polarization, XPD, channel modeling.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) antenna systems have been known for a number of years as a solution for exploiting the spatial diversity and increasing data rates [1]. One major issue of MIMO systems is the need for large inter-antenna spacing to obtain sufficiently low inter-antenna correlation. Cross-polarized MIMO systems are being considered as a solution to design compact antenna systems. By using orthogonally polarized, co-located antennas, low inter-antenna correlation is obtained while maintaining equipment size low [2–7].

In polarized MIMO systems, a linearly polarized wave will leak into the orthogonal polarization due to three mechanisms. The first depolarizing mechanism is due to antenna effects. Although antennas are usually designed to transmit or receive on a given polarization, the are also sensitive to the orthogonal polarization. This sensitivity is represented by the co- and cross-polar antenna pattern [8]. The second mechanism is antenna array tilting [9]. When changing the receiver orientation and keeping the transmitter orientation fixed, a polarization mismatch will occur and change the crosstalk between the different polarizations. The third mechanism is channel depolarization, that occurs due to interactions of the electromagnetic wave with its surroundings, changing the polarization of the waves. The channel depolarization is usually represented by the cross-polar discrimination (XPD), that is defined as the power ratio between the different elements of the MIMO channel matrix. In this paper, only this third mechanism will be investigated.

This work is supported by the Belgian National Scientific Foundation (FRS-FNRS).

Previous measurements have argued that the XPD has a log-normal distribution, but no physical explanation has been given yet for this behaviour [10–12]. The aim of this paper is to investigate small-scale variations of the XPD, by starting from the definition of Rayleigh and Rice fading channels. Measurements have been done in an indoor environment at 3.5 GHz to compare the obtained theoretical model with experimental results by mean of various comparison metrics. The paper is organized as follows: sections II-A and II-B show the derivation of the model for Rayleigh and Rice fading channels respectively. Section II-C highlights the implications of the obtained model. Finally section III compares the model with experimental results.

II. Derivation of the XPD model

A. Rayleigh fading

Without loss of generality, a 1×2 system with a vertical transmitter and a dual-polarized vertical-horizontal receiver will be considered. In that case, the channel matrix is given by

$$\mathbf{H} = \begin{bmatrix} h_{VV} \\ h_{HV} \end{bmatrix} \tag{1}$$

In the case of Rayleigh fading, the amplitude distribution is given by:

$$|h_{VV}| \sim \text{Rayl}\,(\sigma) \tag{2}$$

$$|h_{HV}| \sim \text{Rayl}\,(\alpha\sigma) \tag{3}$$

where σ and $\alpha\sigma$ are the Rayleigh parameters of the VV link and the HV link respectively (in this notation V stands for the vertical polarization, H stands for the horizontal polarization, and XY means that transmitter has polarization Y and receiver has polarization X). Usually, the cross-polar component is smaller than the co-polar component ($|h_{HV}|$ is smaller than $|h_{VV}|$). The parameter α represents which fraction of the power of the transmit polarization is transferred to the perpendicular polarization, and is comprised between 0 and 1. The scattering process (which can be rough-surface scattering, diffraction, etc.) introduces a random phase shift between the different polarization components, so that both elements of $\mathbf{H}$ are independent. If a random variable has a Rayleigh distribution with parameter $\sigma = 1$, the square of this random variable has a chi-square distribution χ^2 with 2 degrees of freedom [13]. Therefore it can be written that:

$$\frac{|h_{VV}|^2}{\sigma^2} \sim \chi_2^2 \quad (4)$$

$$\frac{|h_{HV}|^2}{\alpha^2\sigma^2} \sim \chi_2^2 \quad (5)$$

The XPD is defined as the power ratio of the different elements of the channel matrix:

$$\text{XPD} = |h_{VV}|^2/|h_{HV}|^2 \quad (6)$$

The ratio of two independent chi-square distributed random variables has an F-distribution with the corresponding degrees of freedom [14]. The distribution of the XPD is then given by:

$$\alpha^2\text{XPD}_{\text{lin}} \sim F(2,2) \quad (7)$$

where "‘lin"’ stands for the linear scale.

B. Ricean fading

In the case of Ricean fading channels, the distribution of the amplitudes of h_{VV} and h_{HV} are given by:

$$|h_{VV}| \sim \text{Rice}(\sigma, \nu) \quad (8)$$

$$|h_{HV}| \sim \text{Rice}(\alpha\sigma, \alpha_0\nu) \quad (9)$$

where σ and ν are the Rice parameters of the VV link (i.e. $2\sigma^2$ is the power of the scattered component and ν is the amplitude of the coherent contribution), and $\alpha\sigma$ and $\alpha_0\nu$ are the Ricean parameters of the HV link. Similarly to the Rayleigh case, due to the scattering process both elements of $\mathbf{H}$ are independent. The square of a Rice distributed random variable with parameters $(1, \nu)$ is a non-central chi-square distributed variable with 2 degrees of freedom and non-centrality parameter ν. Therefore we can write:

$$\frac{|h_{VV}|^2}{\sigma^2} \sim \chi_2'^2\left(\frac{\nu^2}{\sigma^2}\right) \quad (10)$$

$$\frac{|h_{HV}|^2}{\alpha^2\sigma^2} \sim \chi_2'^2\left(\frac{\alpha_0^2\nu^2}{\alpha^2\sigma^2}\right) \quad (11)$$

If $X_1 \sim \chi_{\omega_1}'^2(\lambda_1)$, $X_2 \sim \chi_{\omega_2}'^2(\lambda_2)$ and X_1 and X_2 are independent, the random variable $Y = \frac{X_1/\omega_1}{X_2/\omega_2}$ has a doubly non-central F-distribution F'' with degrees of freedom ω_1 and ω_2 and non-centrality parameters λ_1 and λ_2 [14]. Therefore, an exact expression exists for the distribution of the XPD (in linear scale):

$$\alpha^2\text{XPD}_{\text{lin}} \sim F''(2, 2, 2K_{VV}, 2K_{HV}) \quad (12)$$

where $K_{VV} = \nu^2/2\sigma^2$ and $K_{HV} = \alpha_0^2\nu^2/2\alpha^2\sigma^2$ are the Ricean K-factors for the VV and the HV links respectively. Note that (7) is a particular case of (12): if both K-factors are equal to zeros, the doubly-non central F-distribution simplifies to a central F-distribution.

C. Consequences of the F-distributed XPD

Interestingly, it is possible to approximate the doubly non-central F-distribution with degrees of freedom (2,2) by a log-normal distribution (see Figure 1). This is most probably why most results in literature report log-normal distributions for the small-scale variations of the XPD. The standard deviation of the corresponding normal distribution (i.e. when the XPD is expressed in dB) ranges from 0 to 7.8 dB, depending on the value of the non-centrality parameters of the doubly non-central F-distribution, i.e. on the Ricean K-factors.

In the case of Rayleigh fading, it can be seen in (7) that there is no parameter for the F-distribution. In that case, the distribution of the XPD is fixed around its mean value $10\log_{10}(\alpha^2)$ and has a standard deviation of 7.8 dB, as can be seen in Figure 1.

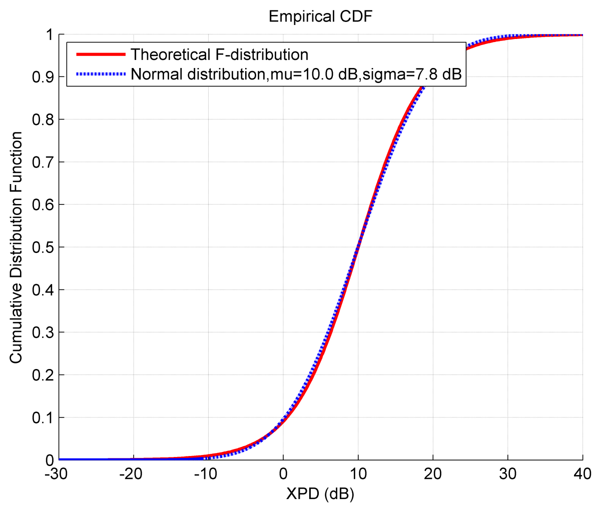

Fig. 1. CDF of F-model and log-normal fitting.

Another particular case is the case where K_{HV} is equal to zero (e.g. in the case of a direct Line-of-Sight with a vertically polarized direct wave). In that case, the doubly non-central distribution simplifies to a singly non-central distribution with degrees of freedom $(2,2)$ and non-centrality parameter $2K_{VV}$:

$$\alpha^2\text{XPD}_{\text{lin}} \sim F'(2, 2, 2K_{VV}) \quad (13)$$

Finally, it is interesting to notice that equation (12) illustrates the relationship of the small-scale variations of the XPD with the Ricean K-factors. Since this relation is non-trivial, equation (12) can be used as a validation tool to validate channel models that include polarization effects.

III. Comparison with Measurements

A. Measurement Setup

Measurements were carried out in an indoor environment to validate the XPD model. A vertical log-periodic antenna was used at the transmitter, and a tri-pole antenna (made of 3 perpendicular monopole antennas) was used at the receiver, of which two antennas were selected to create a vertical-horizontal dual-polarized array. A Rhode&Schwarz ZVA-24 four-port vectorial network analyzer (VNA) was used to measure the channel frequency response on all three receive antennas simultaneously. The carrier frequency was 3.6 GHz with a bandwidth of 200 MHz. The parameters of the VNA are given in Table I. The environment was composed of a main room with surrounding smaller offices, as shown in figure 2.

Center frequency	3.6 GHz
Measured Bandwidth	200 MHz
Frequency step	2 MHz
Dynamic Range	$>$ 30 dB
Number of snapshots	5

TABLE I
VNA parameter settings.

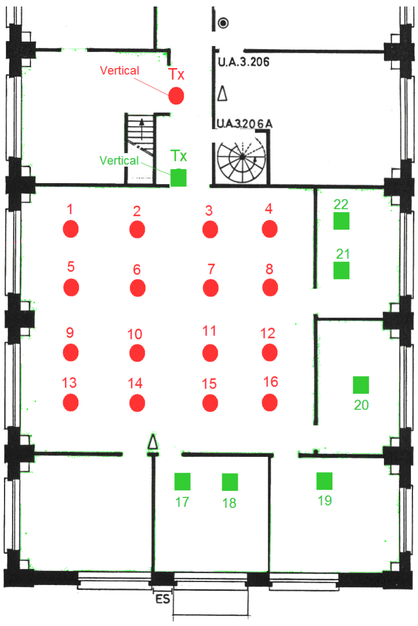

Fig. 2. Floor plan of the measurements.

The transmitter was placed in the corridor leading to the main room, and 22 measurement locations were considered in the main room and in the offices. At each location, a 200-point measurement grid was measured. These 200 points were used to determine the Ricean parameters of the VV-link and the HV-link with a maximum likelihood estimator (MLE). The parameter α is determined with the following straightforward relationship:

$$\alpha = \sigma_{HV}/\sigma_{VV} \tag{14}$$

where σ_{HV} and σ_{VV} are the Ricean σ-parameters of the HV and the VV link respectively.

B. Measurement Results

The measured channel components ($|h_{VV}|$ and $|h_{HV}|$) show Ricean fading for both channel components. An overview of the different values of the K-factors are given in Table II.

Min(K_{VV})	-43.24 dB	Min(K_{HV})	-31.07 dB
Max(K_{VV})	5.82 dB	Max(K_{HV})	7.78 dB
Mean(K_{VV})	1.06 dB	Mean(K_{HV})	0.89 dB

TABLE II
Measured K-factors.

The theoretical XPD variations are obtained by filling in α, K_{VV} and K_{HV} in equation (12). Examples of comparison between experimental data and theoretical model are given in Figures 3 and 4. Note that for visual convenience, the XPD is given in dB value. It can be seen that good agreement is obtained between the experimental cumulative distribution function (CDF) and the doubly non-central F-distribution.

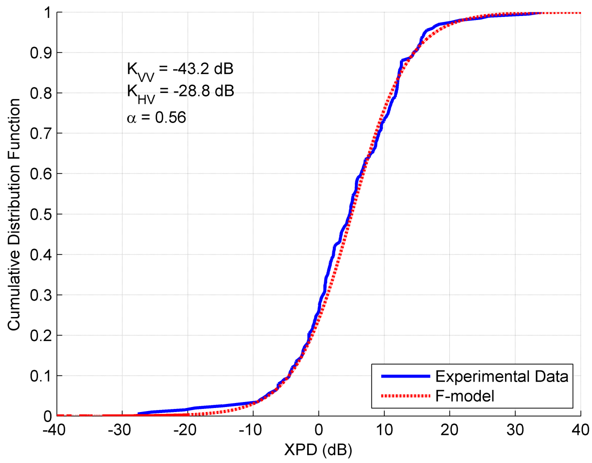

Fig. 3. Cumulative distribution functions of experimental data and F-distribution. ($K_{VV} = -43.2$ dB, $K_{HV} = -28.8$ dB, $\alpha = 0.56$)

The mean square error (MSE) between the experimental CDF and the doubly non-central CDF has been computed for all 22 measurement locations. Table III provides an overview of the results, and confirms the good agreement between experimental results and our theoretical model for all measurement locations. Finally, to confirm the good agreement between the experimental data and the theoretical model, a Kolmogorov-Smirnov test (KS-test) has been carried out for each measurement location. The KS-test compares the experimental cumulative distribution with the theoretical cumulative distribution function, and is sensitive to both the shape and the

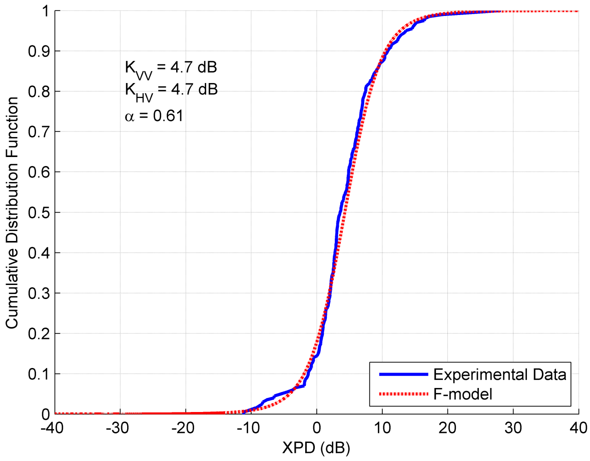

Fig. 4. Cumulative distribution functions of experimental data and F-distribution. ($K_{VV} = 4.7$ dB, $K_{HV} = 4.7$ dB, $\alpha = 0.61$)

Min(MSE)	Max(MSE)	Mean(MSE)
$0.091e-3$	$6.1e-3$	$0.98e-3$

TABLE III
MSE BETWEEN MEASUREMENTS AND MODEL

location of the experimental CDF. The null hypothesis H_0 for the KS-test is that the experimental data and the theoretical model have the same distribution. The alternate hypothesis H_1 is that they do not have the same distribution. In this case, the significance level is set to 5%. The occurrences of H_0 and H_1 are given in Table IV. The KS-test also enables to compare the experimental data with a log-normal distribution. The results of Table IV show that the the theoretical F-model (12) performs slightly better than when fitting the data with a log-normal distribution.

	F-model (12)	Log-normal distribution
Number of occurrences of H_0	21	20
Number of occurrences of H_1	1	2

TABLE IV
RESULTS OF KS-TEST FOR THE 22 MEASUREMENT LOCATIONS.

IV. CONCLUSION

In this paper, it is shown that the small-scale variations of the XPD follow an F-distribution and a doubly non-central F-distributions for Rayleigh and Ricean fading channels respectively. These distributions can be approximated by a log-normal distribution that is traditionally used to describe the XPD variations. The new distribution shows that in the Rayleigh case, the standard variations of the XPD around their mean are fixed and have a value of 7.8 dB. In the Ricean case, the doubly non-central F-distribution provides a direct relationship between Ricean K-factors and XPD variations. Comparison with measurements is given and good agreement is obtained between the derived model and indoor measurements at 3.6 GHz.

ACKNOWLEDGEMENT

This work was carried out in the framework of COST 2100 Action *Pervasive Mobile Ambient Wireless Communications*. The authors would like to thank Pascal Simon, Isabelle Huynen and René Meys for providing all the bits and pieces that were necessary to make the measurements possible. François Quitin and Claude Oestges also acknowledge the financial support of the Belgian Fonds de la Recherche Scientifique-FNRS (FRS-FNRS).

REFERENCES

[1] G. Foschini and M. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, 1998.

[2] B. Lindmark and M. Nilsson, "On the available diversity gain from different dual-polarized antennas," *IEEE J. Select. Areas Commun.*, vol. 19, no. 2, pp. 287–294, 2001.

[3] P. Kyritsi, D. Cox, R. Valenzuela, and P. Wolniansky, "Effect of antenna polarization on the capacity of a multiple element systems in an indoor environment," *IEEE J. Select. Areas Commun.*, vol. 20, no. 6, pp. 1227–1239, 2002.

[4] V. Anreddy and M. Ingram, "Capacity of measured ricean and rayleigh indoor mimo channels at 2.4 ghz with polarization and spatial diversity," in *IEEE Wireless Communications and Networking Conference*, 2006.

[5] M. Andrews, P. Mitra, and R. deCarvalho, "Tripling the capacity of wireless communications using electromagnetic polarization," *Letters to Nature*, vol. 409, pp. 316–318, 2001.

[6] L. Lukama, K. Konstantinou, and D. Edwards, "Performance of a three-branch orthogonal polarization diversity scheme," in *Vehicular Technology Conference 2001 Fall*, vol. 4, 2001, pp. 2033–2037.

[7] V. Erceg, P. Soma, D. Baum, and S. Catreux, "Multiple-input multiple-output fixed wireless radio channel measurements and modeling using dual-polarized antennas at 2.5 ghz," *IEEE Trans. Wireless Commun.*, vol. 3, pp. 2288–2298, 2004.

[8] C. Balanis, *Antenna Theory: analysis and design.* New York, NY, Wiley, 2005.

[9] F. Quitin, C. Oestges, F. Horlin, and P. De Doncker, "Analytical model and experimental validation of cross-polar ratio in polarized MIMO channels," in *19th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, 2008.

[10] G. Calcev, D. Chizhik, B. Göransson, S. Howard, H. Huang, A. Kogiantis, A. Molisch, A. Moustakas, D. Reed, and H. Xu, "A wideband spatial channel model for system-wide simulations," *IEEE Trans. Veh. Technol.*, vol. 56, pp. 389–403, 2007.

[11] M. Shafi, M. Zhang, A. Moustakas, P. Smith, A. Molisch, F. Tufvesson, and S. Simon, "Polarized MIMO channels in 3-D: Models, measurements and mutual information," *IEEE J. Select. Areas Commun.*, vol. 24, no. 3, pp. 514–527, 2006.
[12] C. Oestges, V. Erceg, and A. Paulraj, "Propagation modelling of MIMO mutlipolarized fixed wireless channels," *IEEE Trans. Veh. Technol.*, vol. 53, no. 3, pp. 644–654, 2004.
[13] L. Schmetterer, *Introduction to Mathematical Statistics*. Springer-Verlag, Berlin, Heidelberg, New York, 1974.
[14] R. Chattamvelli, "On the doubly noncentral F distribution," *Computational Statistics & Data Analysis*, vol. Vol. 20, pp. pp. 481–489, 1995.